Formal verification of nonlinear hybrid systems using Ariadne

Tiziano Villa and Luca Geretti

University of Verona

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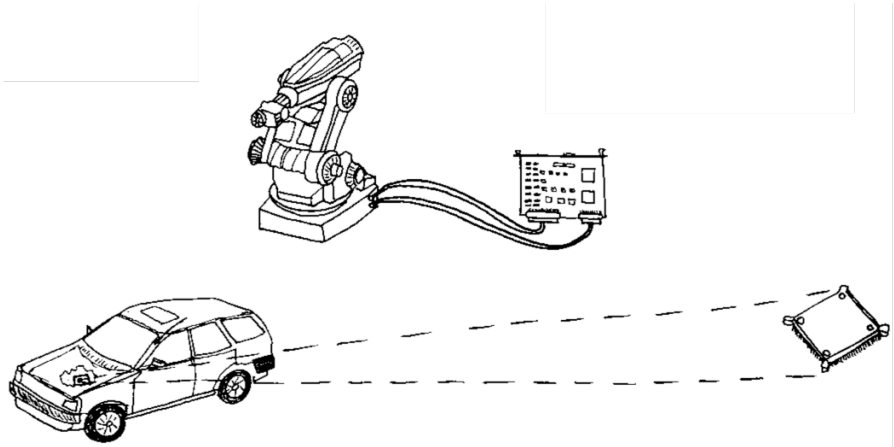
Hybrid systems and nonlinearity

Why consider systems as hybrid?

Real systems composed of a controller with switching *discrete states* and an environment with evolving *continuous variables*.



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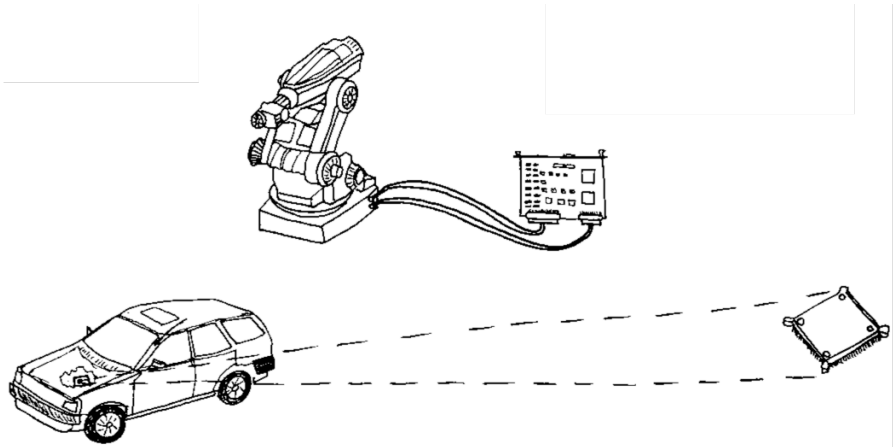

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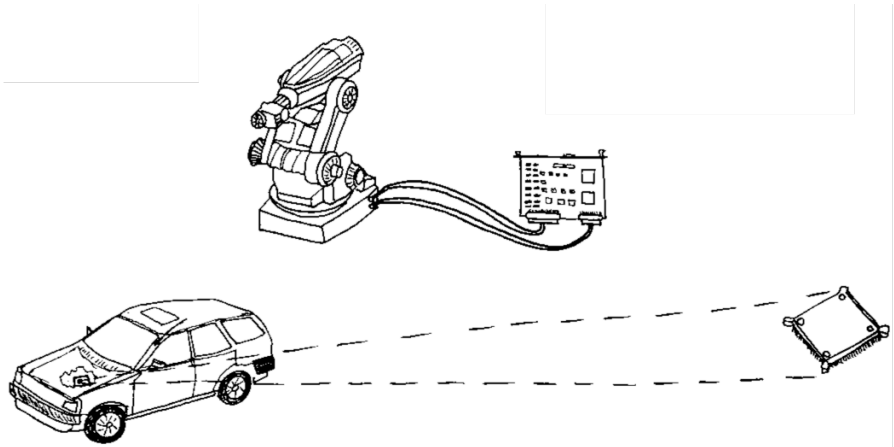

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In general, for our *formal* results we want to be as faithful to the real system as possible

Reachability analysis

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Re can be approximated, but not in *both* an effective and efficient way.

- Some operations on accurate representations are still undecidable.
- Coarse approximations are problematic in terms of reliability of results.

How to approximate regions

Possible choices of approximating Re:

1. **Inner approximation I:** Re strictly contains I.
2. **Outer approximation O:** Re is strictly contained in O (an over-approximation of Re).
3. **ε-lower approximation L_ε:** every point of L_ε is at a distance less than ε from Re (an over-approximation of a subset of Re).

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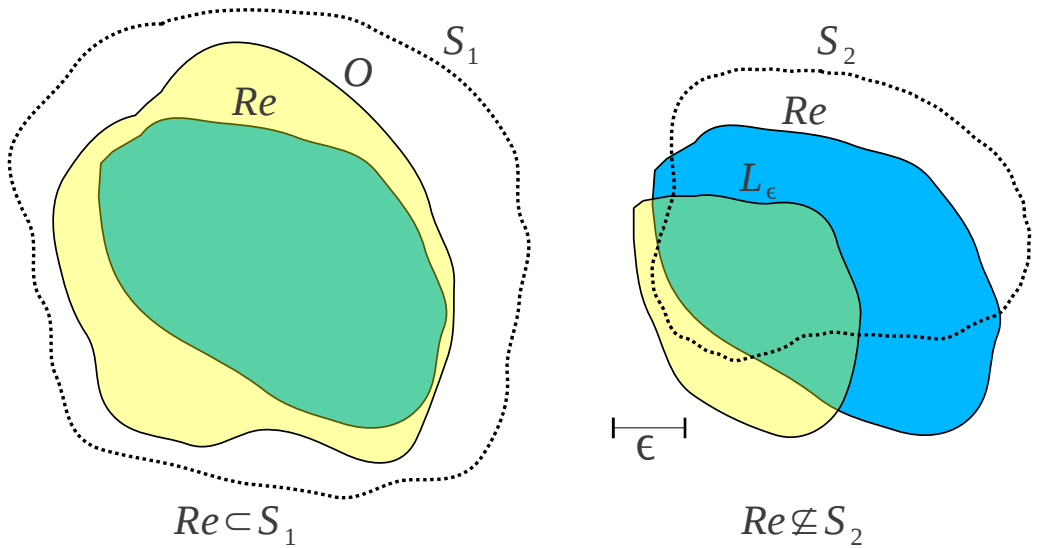
- Inner approximation is not computable in general.
- Outer and ε-lower approximations can be used to verify/falsify properties.

Property satisfaction in terms of sets



- Re is the reachable set, which is unobservable.
- S_1, S_2 are sets satisfying given properties.
- O is the outer approximation of Re.
- L_ε is the ε-lower approximation of Re.

A summary of ARIADNE

- Developed by a joint team including the University of Verona, the University of Maastricht, the University of Padova, and the University of Barry (Florida)
- Uses the formalism of hybrid automata to describe nonlinear time-continuous systems.
- Based on rigorous semantics paired with interval arithmetics to guarantee correctness of verification over approximated sets.
- Written as a C++ library, released as an open source distribution: `http://www.ariadne-cps.org`

Semantics used for evolution

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When numerical inaccuracies make the transition undecidable, all possible choices are taken. Computed sets do not need to include a point of Re.

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Lower semantics

When numerical inaccuracies make the transition undecidable, evolution stops. Computed sets must include at least one point of Re.

Reachable set approximations available

ARIADNE can compute the following approximations of the reachable set (available semantics under parentheses):

- An over-approximated subset, up to a given time t : for proving/disproving properties where a bound on the evolution time is identified [upper, lower];
- An outer approximation : for proving properties using infinite-time evolution [upper];
- For a given $\varepsilon > 0$, an ε-lower approximation : for disproving properties using infinite-time evolution [lower].

Some verification procedures available

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Verify whether a system satisfies some given safety properties.

Dominance

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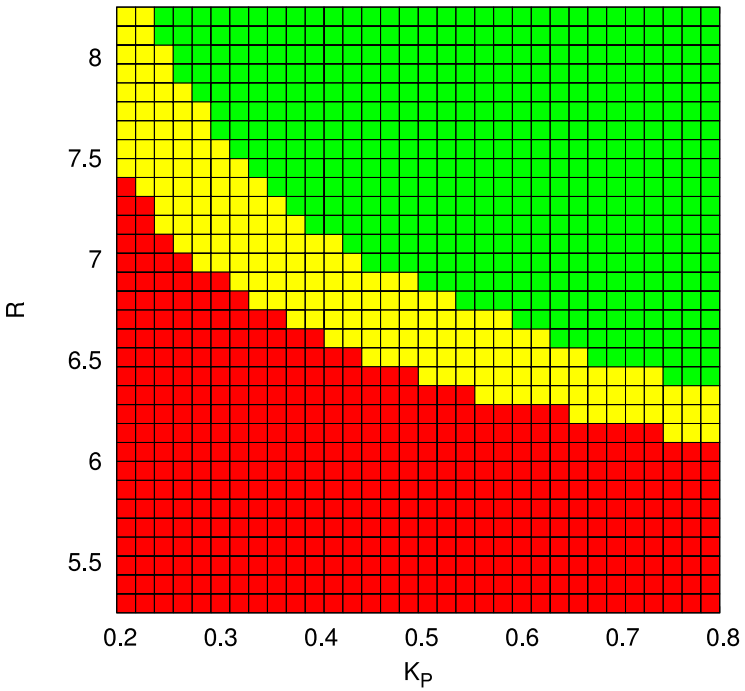
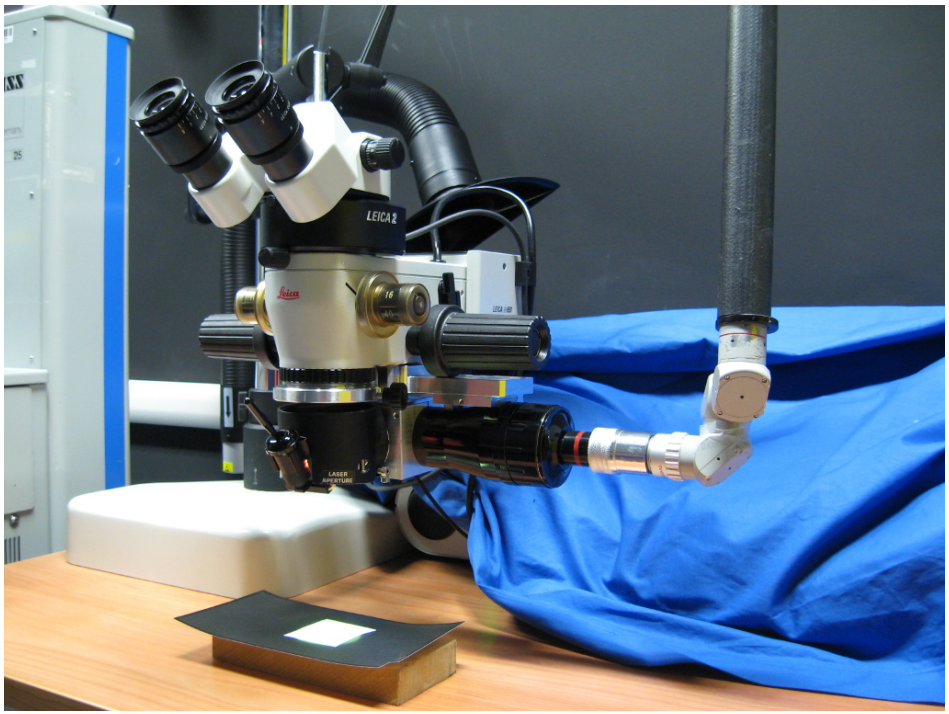


Both these verifications can be performed parametrically by analyzing intervals of constants and splitting the resulting parameters space.

Current field of application: robotic surgery

- Very strict safety requirements.
- Increasing reliance on assisted control for improved accuracy.
- Traditionally focused on control theory specifications, recently adopting formal verification approaches.



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Implement differential inclusions

Allow to model noisy inputs, thus expanding the validity of verification results. Also required to model unspecified inputs, enabling the verification of subsystems in partial isolation

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Expand the verification/automation routines

For example:

- controller synthesis starting from desired properties
- contract-based design for modular systems